With little effort it can be deduced from Eqs. (1, 10, and 11) that for arbitrary values of $M_{aL}^2\beta^2$, the circulation can maintain its initial solid-body distribution and satisfy the boundary conditions only for the trivial case of a constant-area tube and constant velocity on the axis.

Although this conclusion invalidates the analyses of Refs. 1–3 for arbitrary values of swirl, the assumptions of the circulation remaining solid-body rotation and the streamlines tapering uniformly may still be useful when the swirl is sufficiently small.

The effect of swirl on the nozzle's area distribution and performance can be found from a simple analysis if it is assumed that the swirl is small throughout the nozzle and that the Mach number on the axis is given. The precise assumption is that $M_{aL}^2\beta^2$ is small.

Executing the perturbation scheme results in

$$\frac{W}{W_{aL}} = \frac{M_a\Gamma_{aL}^{1/2}}{M_{aL}\Gamma_a^{1/2}}\left[1 + M_{aL}^2\beta^2 \frac{\bar{r}^2}{r_{e0}^2}\frac{\Gamma_a}{\Gamma_{aL}}\frac{(r_{e0}^2 - 1)}{M_a^2}\right] \quad (12)$$

$$\frac{\rho}{\rho_t} = \frac{1}{\Gamma_a^{1/(\gamma-1)}}\left[1 + \frac{M_{aL}^2\beta^2}{2}\frac{\Gamma_a}{\Gamma_{aL}}\frac{\bar{r}^2}{r_{e0}^2}\right] \quad (13)$$

$$I = I_0\bar{r}^4\left(1 + \frac{M_{aL}^2}{2}\beta^2(1 - \bar{r}^2) \times \left\{1 - \frac{\Gamma_a}{\Gamma_{aL}r_{e0}^2}\left[1 + \frac{2(r_{e0}^2 - 1)}{M_a^2}\right]\right\}\right) \quad (14)$$

$$\frac{H}{a_t^2/(\gamma - 1)} = 1 + (\gamma - 1)\frac{M_{aL}^2\beta^2}{\Gamma_{aL}}\bar{r}^2 \times \left(1 + \frac{M_{aL}^2\beta^2}{4}(1 - \bar{r}^2)\left\{1 - \frac{\Gamma_a}{r_{e0}^2\Gamma_{aL}} \times \left[1 + \frac{2(r_{e0}^2 - 1)}{M_a^2}\right]\right\}\right) \quad (15)$$

$$r_e^2 = r_{e0}^2\left(1 + \frac{M_{aL}^2\beta^2}{4}\left\{1 - \frac{\Gamma_a}{r_{e0}^2\Gamma_{aL}} \times \left[1 + \frac{2(r_{e0}^2 - 1)}{M_a^2}\right]\right\}\right) \quad (16)$$

Note that, for M_a large, the density distribution is independent of swirl.

The values of M_a^2, which occur at the throat, are obtained by determining the condition for maximum total mass flow at the throat. This is a choking criterion in the spirit of a one-dimensional analysis, and is believed to be consistent in the framework of the present analysis. This criterion results in

$$M_{a*}^2 = 1 + (\gamma + 1)M_{aL}^2\beta^2/8r_{e*}^2[3 + \gamma - 4r_{e*}^2] \quad (17)$$

which shows that $M_{a*}^2 > 1$, and this does not agree with Refs. 1–3. The reason for this discrepancy is that various choking criterions and the various assumptions have been used by the different authors. However, this result is obtained from a consistent analysis and is believed to be correct.

The mass flux through the nozzle provides an interesting comparison with the zero swirl case. If it is assumed that the zero swirl nozzle operates at the same total conditions that exist on the axis of the nozzle with swirl, and that they have the same initial Mach number and area, then the one with swirl will have the larger mass flow because its average pressure is larger; the resulting formula is

$$Q/Q_0 = [1 + (M_{aL}^2\beta^2/4)] \quad (18)$$

However, if the zero swirl nozzle operates at the total conditions that exist at the edge of the subsonic section, the swirl nozzle has less flow because its average pressure would be smaller than the zero swirl case. This flux can be shown to be

$$\frac{Q}{Q_0} = \left\{1 + \frac{M_{aL}^2\beta^2}{4}\left[1 - \frac{2(\gamma + 1)}{(\gamma - 1)\Gamma_{aL}}\right]\right\} \quad (19)$$

These results give sufficient information for the calculation of additional nozzle performance parameters.

Concluding Remarks

The quasi-cylindrical approximation results in a simplified theory with enough features of the exact equations to yield a meaningful first approximation of exact solutions.

This theory shows for arbitrary magnitudes of swirl that the initial distribution of the circulation cannot be maintained throughout the nozzle, except for the case of constant swirl. Furthermore, the assumption that the circulation retains its initial distribution and that the streamlines contract uniformly is only valid as a first approximation for the special case of small swirl.

References

[1] Bastress, E. K., "Interior ballistics of spinning solid-propellant rockets," J. Spacecraft Rockets **2**, 455–457 (1965).

[2] Manda, L., "Spin effects on rocket nozzle performance," J. Spacecraft Rockets **3**, 1695–1696 (1966).

[3] King, M. L., "Comment on 'Spin effects on rocket nozzle performance,'" J. Spacecraft Rockets **3**, 1812–1813 (1966).

[4] Manda, L. J., "Reply by author to M. King," J. Spacecraft Rockets **3**, 1813–1814 (1966).

[5] King, W. S., "A theoretical investigation of swirling flows through a nozzle," Ph.D. dissertation, Univ. of California at Los Angeles (1967).

[6] Hall, M. G., "The structure of concentrated vortex cores," *Progress in Aeronautical Sciences*, edited by D. Kuchemann, (Pergamon Press Inc., New York, 1966), Vol. 7, p. 53.

[7] Mager, A., "Approximate solution of isentropic swirling flow through a nozzle," Am. Rocket Soc. J. **31**, 1140–1148 (1961).

Comment on "Study of Electric Propulsion for Manned Mars Missions"

Charles L. Zola*

NASA Lewis Research Center, Cleveland, Ohio

The purpose of this comment is to correct a part of a recent paper by Ragsac.[1] In Fig. 1 of Ragsac's paper, a comparison is made between the Zola[2,3] characteristic length method and exact data by Melbourne.[4] The comparison is made using the parameter

$$J = \frac{1}{2}\int_0^T a^2 dt$$

for a variety of Earth-Mars transfers. Unfortunately, the data given for Zola's method contain some apparent numerical errors and lead Ragsac to conclude that this method is in error for transfer times beyond 180 days.

Figure 1 of this comment shows a corrected evaluation of the length method using the more usual parameter

$$J = \int_0^T a^2 dt$$

In this figure, the approximate values of J are calculated from the sum of the hyperbolic excess velocities ΔV_{HT} required at Earth and Mars for single-conic, high-thrust, heliocentric

Received June 6, 1967.

* Aerospace Engineer. Member AIAA.

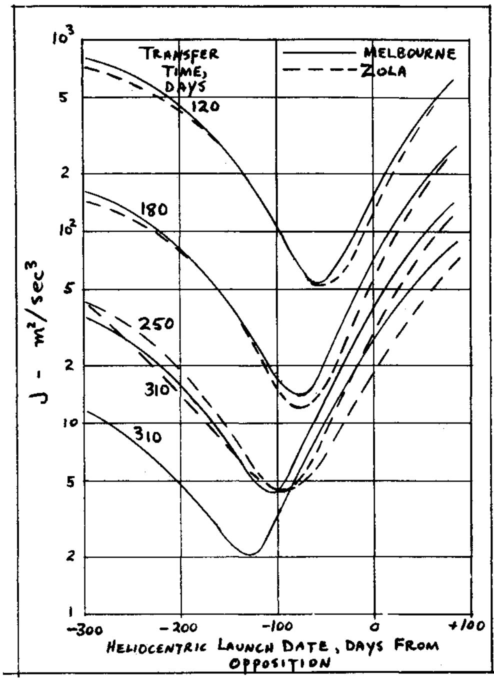

Fig. 1 Characteristic length (Zola) results compared with Melbourne's data for Earth-Mars transfers.

transfers at each transfer time and polar travel angle. The equations used for these calculations are

$$L = T\,\Delta V_{HT}/2 \tag{1}$$

$$J = 12L^2/T^3 \tag{2}$$

where L is the characteristic length and T is the actual transfer time. A more detailed description of this method is given in Refs. 2 and 3. The corrected figure shows that the error in the length method does not grow rapidly until the transfer time exceeds the Hohmann time (~260 days). This is because the single-conic, high-thrust transfers used in this data do not give minimum ΔV_{HT} beyond the Hohmann time.

The characteristic length correlation is a method of predicting the performance of one mode of rocket operation from known performance of some other mode. One use, if desired, is predicting variable thrust data from high-thrust data, as in Fig. 1 of this comment, but its primary purpose is to help analyze the performance of low-thrust spacecraft with constant thrust and specific impulse. Reference data for the correlation can be precalculated solutions of the trajectory problem using constant-thrust, variable-thrust, or high-thrust operation.

The characteristic length method has many accuracy limitations that are discussed in Ref. 3. Also noted in Ref. 3 is that low-thrust data at constant thrust and specific impulse are better predicted with variable-thrust data as a reference, not the single-conic, high-thrust data used in Fig. 1. Furthermore, as noted by Ragsac,[1] recent computing methods make variable-thrust solutions obtainable in a matter of seconds, if not already available in published form.

Although the accuracy of the length correlation may vary from mission to mission, it is most accurate for Mars and Venus orbiter trajectories. Also, MacKay[5] et al. and Brown[6] have successfully used the method for constant low-thrust mission studies which include combined high- and low-thrust operation. The effect of combined thrust operation is estimated by assuming that the high-thrust maneuvers at either end of the path reduce the required ΔV_{HT}. The reduced ΔV_{HT} means lower L and leads to lower propellant and power for the low-thrust spacecraft. MacKay[5] et al. have found this method to be accurate to within 5% of exact calculations for required initial orbiting mass for Earth-Mars round trip missions.

References

[1] Ragsac, R. V., "Study of electric propulsion for manned Mars missions," J. Spacecraft Rockets **4,** 462–468 (1967).

[2] Zola, C. L., "Trajectory methods in mission analysis for low-thrust vehicles," AIAA Paper 64–51 (1964).

[3] Zola, C. L., "A method of approximating propellant requirements of low-thrust trajectories," NASA TN D-3400 (1966).

[4] Melbourne, W. G. and Sauer, C. G., "Optimum Earth-to-Mars round trip trajectories utilizing low-thrust power-limited propulsion systems," Jet Propulsion Lab. TR 32-376 (March 1963).

[5] MacKay, J. S. et al., "Manned Mars landing missions using electric propulsion," NASA TN D-3194 (1966).

[6] Brown, H. and Coates, G. L., "Application of nuclear electric propulsion to manned Mars missions," J. Spacecraft Rockets **3,** 1402–1408 (1966).

Reply by Author to C. L. Zola

R. V. Ragsac*

United Aircraft Research Laboratories, East Hartford, Conn.

In the comment by Zola, it is stated that the comparison in Fig. 1 of Ref. 1 uses the parameter $J = \frac{1}{2}\int a^2dt$. However, the parameter employed is $J = \int a^2dt$; this fact is noted[1] where Fig. 1 is first cited. The curve for the 250-day transfer time (variable thrust) was carefully checked using the J from Eqs. (1) and (2) of Zola's comment and the hyperbolic excess speeds listed in the NASA Planetary Flight Handbook, SP-35. As required by Zola's method,[2] the reference-mode solution must correspond to the same trajectory problem in the new mode. Since comparisons are being made for heliocentric transfer trajectories between planets moving in mutually inclined, elliptic orbits (i.e., three-dimensional), the corresponding hyperbolic excess speeds from SP-35 were used. The results are shown in Fig. 1. The use of more data points uncovers a discontinuity not previously noted. Because of the three-dimensional effects, the hyperbolic excess speed values at certain planetary configurations (launch dates) rise steeply and then descend, forming a "ridge."[4] Consequently the effect on J should be quite similar. Other than this ridge, the curve remains essentially the same as originally presented in Ref. 1.

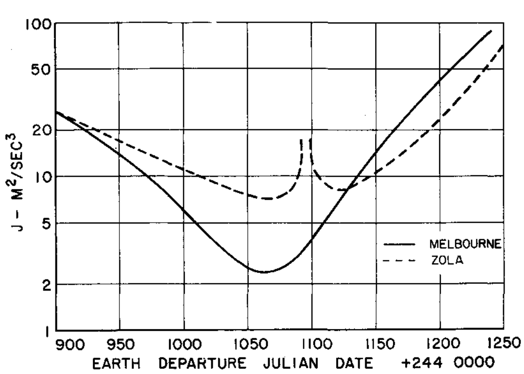

Fig. 1 Characteristic length (Zola) results compared to Melbourne's data for 250-day Earth-Mars trip.

Received July 6, 1967; revision received July 19, 1967.

* Senior Research Engineer. Member AIAA.